Study on the bubble transport mechanism in an acoustic standing wave field

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Abstract

The use of bubbles in applications such as surface chemistry, drug delivery, and ultrasonic cleaning etc. has been enormously popular in the past two decades. It has been recognized that acoustically-driven bubbles can be used to disturb the flow field near a boundary in order to accelerate physical or chemical reactions on the surface. The interactions between bubbles and a surface have been studied experimentally and analytically. However, most of the investigations focused on violently oscillating bubbles (also known as cavitation bubble), less attention has been given to understand the interactions between moderately oscillating bubbles and a boundary. Moreover, cavitation bubbles were normally generated in situ by a high intensity laser beam, little experimental work has been carried out to study the translational trajectory of a moderately oscillating bubble in an acoustic field and subsequent interactions with the surface. This paper describes the design of an ultrasonic test cell and explores the mechanism of bubble manipulation within the test cell. The test cell consists of a transducer, a liquid medium and a glass backing plate. The acoustic field within the multi-layered stack was designed in such a way that it was effectively one dimensional. This was then successfully simulated by a one dimensional network model. The model can accurately predict the impedance of the test cell as well as the mode shape (distribution of particle velocity and stress/pressure field) within the whole assembly. The mode shape of the stack was designed so that bubbles can be pushed from their injection point onto a backing glass plate. Bubble radial oscillation was simulated by a modified Keller–Miksis equation and bubble translational motion was derived from an equation obtained by applying Newton's second law to a bubble in a liquid medium. Results indicated that the bubble trajectory depends on the acoustic pressure amplitude and initial bubble size: an increase of pressure amplitude or a decrease of bubble size forces bubbles larger than their resonant size to arrive at the target plate at lower heights, while the trajectories of smaller bubbles are less influenced by these factors. The test cell is also suitable for testing the effects of drag force on the bubble motion and for studying the bubble behavior near a surface.

1. Introduction

The understanding of interactions between a boundary and air bubbles in aqueous solutions is of particularly industrial interest since ultrasound-driven bubbles have substantial physical and chemical effects on the target surface [1]. It is well-known that the forces generated from cavitation bubbles are sufficient to erode metal surfaces [2], clean contaminated wafers [3–5], and induce cell lysis or cell membrane porosity [6–13]. Numerous investigations on the mechanism of bubble oscillation near a boundary have been carried out for decades, e.g. [14,15]. A comprehensive review of this topic can be found in recent work by Lauterborn and Kurz [16]. Despite the successful studies of cavitation bubble induced physical and chemical effects, less attention has been given to understand the mechanism of interactions between moderately oscillating bubbles and a surface. Moreover, most of the previous studies focused on the collapse and consequent shape distortion of laser-generated bubbles which have already been placed next to a target area. However, the means to effectively move moderately oscillating bubbles to the appointed target in the first place is still open to question. Multiple bubble transport is also preferable in real life applications, but to date there are only few reports on the experimental configuration for this purpose.

To achieve bubble motion control, acoustic systems are typically fabricated based on a layered resonator. The main part of the resonator is a piezoelectric material which is bonded to several matching layers. Acoustical standing waves can be generated in a
liquid layer (matching layer) which is terminated by a reflector. A one-dimensional equivalent network model (1D model) has been widely used for predicting the acoustic responses of such multi-layered structures [17,18]. In the 1D model, characteristics of a sound field including pressure profile and amplitude are calculated based on the properties of the matching layers. At a certain frequency, the mode shape (particle velocity or stress/pressure distribution) of an acoustic field can then be determined. Similar resonator devices have been used by many authors to study particle manipulation [19–22] and cell localization [23–26].

Few researchers, however, have applied the resonator system to investigate bubble motion and control. This is believed to be due to the fact that bubble motion in an acoustic field not only depends on the characteristics of the sound field, but also on the bubble oscillation which complicates the analysis of the bubble movement. A gas bubble driven below its resonance frequency in a weak standing wave field moves towards the pressure anti-node, while a bubble driven above its resonance frequency moves towards the pressure node instead. This effect is attributed to the primary Bjerknes force on a bubble and has been studied extensively by King [27], Yoshioka et al. [28], Eller [29], Crum [30], Lee and Wang [31] and Doinikov [32]. On the other hand, in a high intensity acoustic field, two types of bubble translational instabilities have been recognized. The first one, also known as ‘dancing’ motion, refers to the bubble erratic behaviors when bubbles travel in a sound field. It was first observed by Gaines [33], Strasberg and Benjamin [34] and Eller and Crum [35] and later investigated by Mei and Zhou [36], Feng and Leal [37] and Doinikov [38] in more detail. A generally accepted explanation for this phenomenon attributes the bubble surface modes, which come into existence once the acoustic pressure amplitude exceeds a threshold value, as the main cause. The second type of translational instability results from the fact that the primary Bjerknes force acting on a bubble changes sign at higher acoustic pressure [39,40]. This change is a result of the increased phase shift between bubble volumetric pulsation and the driving pressure. This behavior was reported by Miller [41], Khanna et al. [42], and Kuznetsova et al. [43]. Theoretical investigations were carried out by Abe et al. [44], Watanabe and Kukita [45] and more recently were extended by Doinikov [46] and Mettin and Doinikov [47]. However, there remains a need for an experimental method to be developed to effectively move bubbles to a boundary and excite bubbles near the boundary surface.

The purpose of this study is to describe a bubble transport technique which uses a multi-layered resonator to push bubbles to a reflector surface and examine the bubble behavior when they migrate in the sound field as well as close to the target surface. In Section 2, details of the experimental setup which were used to study the translational and oscillatory motion of bubbles are described. Section 3 provides the theoretical models of bubble motion in an acoustic standing wave field. Comparisons between the theoretical predictions and experimentally obtained results are given in Section 4. A conclusion is provided in Section 5.

2. Theory and experimental setup

As described in the introduction, the means to effectively move bubbles to a region of interest is critical in developing the bubble control technique using acoustic wave. Three tools for the study of bubble transport mechanism and bubble oscillatory motion are presented here. Firstly, a bubble generator which is based on the principle of electrolysis is used. Secondly, a standing wave field is generated by a multi-layered structure and the characteristics of this field are simulated by a 1D model. The 1D model allows us to accurately predict the acoustic responses of the device including impedance and pressure amplitude in the layered structure. Thirdly, details of bubble translational and oscillatory motion are recorded by an optical observation system. Using these three tools, we determined the bubble trajectories within the test cell and studied the bubble oscillation near a surface.

2.1. Bubble generator

Bubbles used in this study are generated by an electrolysis method. Two wires (tin-coated copper) are connected to a DC power supply (TNG 35, Voltcraft, Germany) and the electrical potential is set to 5 V. The free ends of the wires are placed at x = 3 mm (the origin of the coordinate system is set at the center of the transducer-water boundary as shown in Fig. 2). Hydrogen gas bubbles are generated at the tip of the negative electric wire and escape from there afterwards. The bubble diameter varies from 10 μm when there is no ultrasound, up to 200 μm when the ultrasound device is switched on. The large bubbles are the outcomes of bubble coalescence processes which are significantly enhanced in the presence of ultrasound.

2.2. 1D model for acoustic standing wave field

The method implemented in our 1D model is the modified equivalent electrical network of a transducer. In the model, a transducer is treated as a purely electrical circuit and can be analyzed by conventional circuit analysis techniques. The 1D model has been verified to accurately predict acoustic responses of multi-layered structures within the first few resonance frequencies. The model was described in detail by Wilcox et al. [17] and here a short outline is presented. The basic idea of this approach is shown in Fig. 1. The piezoelectric layer is represented as a three-port electrical network which is described by the following matrix equation:

\[
\begin{pmatrix}
  F_1 \\
  F_2 \\
  V_1 \\
\end{pmatrix} = \begin{pmatrix}
  A_{Zpiezo} \csc k_{piezo} d_{piezo} & A_{Zpiezo} \csc k_{piezo} d_{piezo} & h_{11}/\omega \\
  -i A_{Zpiezo} \csc k_{piezo} d_{piezo} & A_{Zpiezo} \csc k_{piezo} d_{piezo} & h_{11}/\omega \\
  1/\omega c_0 & 0 & 1 \\
\end{pmatrix} \begin{pmatrix}
  v_1 \\
  v_2 \\
  I_1 \\
\end{pmatrix}
\]

(1)

where \( F_1 \) and \( v_1 \) (i is the index) are the force and particle velocity at the two acoustic ports. \( V_2 \) and \( I_1 \) are the voltage and current applied to the electric port of the piezoelectric material. \( A, Z_{piezo}, d_{piezo}, \) and \( k_{piezo} \) are the area, characteristic impedance, thickness, and acoustic wave number of the piezoelectric layer respectively. \( h_{11} \) is the transmitting constant of the piezoelectric material in the x direction (longitudinal direction), \( C_0 \) is the clamped (zero strain) capacitance. \( \omega \) is the angular frequency.

For a non-piezoelectric layer, the input force and velocity (\( F_1 \) and \( v_1 \)) are related to the output force and velocity (\( F_2 \) and \( v_2 \)) by a matrix

\[
\begin{pmatrix}
  F_2 \\
  v_2 \\
\end{pmatrix} = \begin{pmatrix}
  \cos k_d d_1 & -i A Z \sin k_k d_1 \\
  -(i/A Z) \sin k_k d_1 & \cos k_k d_1 \\
\end{pmatrix} \begin{pmatrix}
  F_1 \\
  v_1 \\
\end{pmatrix} = T_1 \begin{pmatrix}
  F_1 \\
  v_1 \\
\end{pmatrix}
\]

(2)

where \( Z_d, d_1, \) and \( k_k \) are the characteristic impedance, thickness, and acoustic wave number of the layer respectively. \( A \) is the area of the layer which is the same as the piezoelectric one, \( T_1 \) is the transfer matrix of this non-piezoelectric layer.

As the acoustic generator in Fig. 1a is represented by a number of cascaded two-port (non-piezoelectric layer) networks and a three-port (piezoelectric layer) one, it is possible to calculate the acoustic response of the whole stack by reducing the continuous networks to a single two-port one. The 3 by 3 matrix in Eq. (1) is replaced by an equivalent symmetrical one for the sake of simplicity and the notations in Wilcox’s work are used here [17].
The ratios $F_1/v_1$ and $F_2/v_2$ are represented by $Z_{0B}$ and $Z_{0A}$ respectively, and Eq. (3) can be reduced to a 2 by 2 matrix using $Z_{0B}$.

$$
\begin{pmatrix}
F_1 \\
F_2 \\
V_3
\end{pmatrix}
= 
\begin{pmatrix}
Z_{11} & Z_{12} & Z_{13} \\
Z_{21} & Z_{22} & Z_{23} \\
Z_{31} & Z_{32} & Z_{33}
\end{pmatrix}
\begin{pmatrix}
v_1 \\
v_2 \\
i_3
\end{pmatrix}
$$

The system, the transfer matrix $T_{PZ}$ is pre-multiplied by the $T_{in}$ and the system transfer matrix $T$ is found

$$
T = T_{in} \cdots T_{i2} T_{i1} T_{PZ}
$$

For a transducer, $T$ relates the voltage/current ($V_3$ and $I_3$) of the input signal to the force/velocity ($F_{out}$ and $v_{out}$) in the output medium through Eq. (8).

$$
\begin{pmatrix}
F_{out} \\
v_{out}
\end{pmatrix}
= T
\begin{pmatrix}
V_3 \\
i_3
\end{pmatrix}
$$

2.3. Acoustical standing wave generator

The multi-layered resonator has been widely used in many applications such as particle manipulation, cell separation, and ultrasonic transducer design. An acoustical standing wave field is generally established along the structure (axial direction). The sound field is believed to be uniformly distributed in the radial direction (or directions normal to the axial axis for non-cylindrical shape structure) and this assumption is only valid when the length of the structure is larger than its width and the width is less than half of the standing wave wavelength (the use of isotropic materials is assumed). As the sound field within the multi-layered structure only varies in the axial direction, the 1D model can accurately predict the acoustic responses of such resonators. To clearly illustrate the bubble transport mechanism, it is favorable to design a

![Diagram](image)
simple standing wave field with one pressure node and one pressure anti-node along the axial direction in a liquid medium (matching layer) and with little variations in the radial direction. Within this acoustic field, the bubble migrating direction can be easily categorized either towards the pressure node or pressure anti-node based on bubble size and acoustic pressure amplitude. It is well known that a resonant air bubble in water has a radius of 30 μm at 100 kHz. The bubble generator produces bubbles of about 100 μm (radius) indicating resonance frequencies of about 30 kHz. Therefore, in order to ensure operation above the resonance frequencies of the bubbles a stack with the second resonance frequency of 108 kHz was designed.

A schematic diagram of the setup is shown in Fig. 2. A continuous sinusoidal wave is transmitted from a waveform generator (APG3021, Tektronix, USA) to a transducer via an amplifier (HSA 4101, NF corporation, Japan). The input signal is monitored by an oscilloscope (TDS 220, Tektronix, USA). A standing wave is established in the stack along the x axis and has negligible variations in the y and z directions (see Section 4.1). The origin of the coordinates is set at the center of the transducer-water boundary (x = 0, y = 0, z = 0). A water layer constrained within a brass chamber is placed between the transducer and a glass plate (from x = 0 mm to x = 5 mm). To allow light to pass through the water layer for optical observations, two glass windows are fit on the sides of the brass chamber. Bubbles escape from the wire connected to the negative port of the DC power supply and migrate in the water medium. The wires connected to the DC power supply were positioned at x = 3 mm.

The acoustic standing wave generator consisted of a round transducer, a square liquid (deionized water) layer of 5 mm thickness held in a brass block (5 mm by 5 mm by 5 mm) and a round borosilicate glass plate of 0.1 mm thickness (VWR, UK). To fit the water layer and the bubble injection point at the same time. Recorded videos were transmitted back to a computer and were analyzed by an object tracking program written in Matlab (Mathworks Inc., USA). In the Matlab program, the center of a bubble was tracked in each frame and a plot of the bubble center positions with respect to time was obtained.

To examine the validity of the 1D model, the pressure field in the water layer at 108 kHz was measured by a calibrated needle hydrophone (HPM1/1, Precision Acoustics, UK) and compared with the result obtained from the 1D model. A diagram of the testing setup is shown in Fig. 3. The hydrophone is fixed on a three dimension scanning frame (3 axis motorized scanning system, Time and Precision Industries Ltd, UK) which is controlled by a computer. The hydrophone is powered by a DC power supply (DC3, Precision Acoustics, UK) and is used to measure the pressure profile in the x, y and z directions in the water layer. The pressure amplitudes along the x axis are measured from point A (see Fig. 3) at the center of the base of the water column to point B (see Fig. 3), which is at the center of the top of the water column with a step size of 0.5 mm. The difference between the test cells shown in Fig. 3 and 2 is that the glass plate which is used as a reflector in Fig. 2 is removed in the hydrophone measurement case. It needs to be pointed out that the purpose of the hydrophone measurement is to solely examine the 1D model predictions so that confidence in its validity can be obtained. As the 1D model treats each matching layer in the same way, it is reasonable to accept that the pressure profile of a testing cell with the thin glass plate (sound soft boundary) can be accurately predicted by the 1D model which is verified in the case without the additional glass plate.

2.4. Optical observation system

A high speed camera (FastCam SA5, Photron, USA) was used to investigate the bubble moving trajectory and oscillatory motion. The maximum frame rate of the camera is 1 Mega frames/s and is therefore suitable for analyzing bubble motion in pressure fields oscillating at hundreds of kilo-Hertz. Fig. 4 shows the schematic diagram of the measurement setup. A backing light source was positioned opposite to the high speed camera with the standing wave generator in the middle. A viewing window of 3.9 mm by 3.8 mm was chosen to cover the glass plate and the bubble injection point at the same time. Recorded videos were transmitted back to a computer and were analyzed by an object tracking program written in Matlab (Mathworks Inc., USA). In the Matlab program, the center of a bubble was tracked in each frame and a plot of the bubble center positions with respect to time was obtained. The dimensions of objects in a video were calibrated with a standard 300 μm long stick.

![Fig. 2. A schematic diagram of the acoustic standing wave generator.](image-url)
3. Bubble oscillation and translation models

In this section, details of the theoretical approaches to model bubble oscillation and translation in a sound field are provided. To model the bubble oscillation and translation, the radial and translational equations are derived simultaneously. The radial oscillation of a bubble is represented by a modified Keller–Miksis equation \[ \frac{1}{C_0} \frac{dR_c}{dt} = \frac{R_c}{C_0^2} + \frac{3}{2} \frac{R_c}{C_0^2} - \frac{1}{\rho} \left( 1 + \frac{R_c}{c} \right) P_{sc} - \frac{R}{\rho c} P_{ex} \]

\[ \frac{dx}{dt} = \frac{1}{C_0} \frac{dR_c}{dt} \]

Mettin and Doinikov [47] have pointed out that the missing term is the consequence of the added scattered pressure caused by the translational motion of the bubble.

\[ \left( 1 - \frac{R}{c} \right) \frac{dR_c}{dt} + \left( \frac{3}{2} - \frac{R}{2c} \right) R_c - \frac{1}{\rho} \left( 1 + \frac{R_c}{c} \right) P_{sc} - \frac{R}{\rho c} P_{ex} - \frac{x^2}{4} = 0 \]  

\[ P_{sc} = \left( P_0 + \frac{2\sigma}{R_0} \right) \left( \frac{R_0}{R} \right) - \frac{2\sigma}{R} - \frac{4\eta R}{R} - P_0 - P_{ex} \]
where $R$ is the time-varying radius of a bubble, $R_0$ is the equilibrium radius. $c$ and $\rho$ are the sound velocity and density of the liquid respectively. $P_0$ is the hydrostatic pressure, $\gamma$ is the surface tension, $\gamma$ is the polytropic exponent of the gas within the bubble, $\eta$ is the viscosity of the liquid. $\dot{x}$ is the translational velocity of the bubble in the $x$ axis. $P_{ex}$ is the external driving signal which is defined as a standing wave here:

$$P_{ex} = P_0 \sin(\omega t) \sin(kd)$$  \hfill (11)

where $P_0$ is the pressure amplitude, $\omega$ is the angular frequency, and $k$ is the wave number. As only a one dimensional standing wave is considered here, $d$ is the distance between the center of the bubble and a pressure anti-node along the $x$ axis as shown in Fig. 2.

The translational motion of a bubble is derived based on Newton’s second law, given by

$$\ddot{x} + \frac{3\dot{x}}{R} = \frac{3F_{ex}}{2\pi\rho R^3}$$  \hfill (12)

$$\ddot{y} + \frac{3\dot{y}}{R} = \frac{3F_{ey}}{2\pi\rho R^3}$$  \hfill (13)

where $x$ and $y$ are the positions of the bubble center on the $x$ and $y$ axes as shown in Fig. 2. The overdot denotes the time derivative. In a two dimensional plane, the coordinates of the bubble center are given by the time-varying variables $x$ and $y$. $F_{ex}$ and $F_{ey}$ are the external force in the $x$ axis and $y$ axis directions respectively. In the $x$ axis direction, the $F_{ex}$ is equal to the sum of the primary Bjerknes force ($F_p$) and the viscous drag force ($F_{\eta}$) [50].

Fig. 5. Calculated impedance of test cell without (solid line) and with (dotted line) the glass plate. The frequencies used in the calculation are from 50 kHz to 300 kHz.

Fig. 6. Measured (scatter line) and simulated (solid line) pressure profiles in the water layer at 108 kHz. The pressure distribution is measured from the center of the base of the water column (A) to the center of the top of the water column (B) as shown in Fig. 3. The input signal amplitude is 1.8 V. The solid vertical line on the left side indicates the position ($x = 1.7$ mm) where the $y$ and $z$ direction measurement shown in Fig. 7 was carried out. The dashed vertical line on the right side shows the position of the bubble injection point ($x = 3$ mm).

Fig. 7. Pressure profiles measured by the hydrophone in the $y$ axis and $z$ axis at 108 kHz for input signal of 1.8 V and $x = 1.7$ mm. (a) measured pressure profile in the $y$ axis; (b) measured pressure profile in the $z$ axis.

Fig. 8. A simulated pressure distribution of the multi-layered structure (with the glass layer) at 108 kHz for input signal of 2 V.

Fig. 9. The relationship between bubble resonance frequency and bubble radius. Bubble radius ranges from 10 $\mu$m to 300 $\mu$m.
\[ F_p = -\frac{4\pi}{3}R^3kP_0\sin(\omega t)\cos(\kappa d) \]  
\[ F_{vx} = -12\pi\eta R(\dot{x} - v_x) \]

where \( v_x \) is the liquid velocity that is generated by the imposed acoustic field at the center of the bubble

\[ v_x = \frac{P_0}{\rho c} \cos(\omega t)\cos(\kappa d) \]

In the y axis direction, the \( F_{vy} \) is the sum of buoyancy force (\( F_b \)) and viscous force (\( F_{vy} \)) \[ F_b = \frac{4\pi}{3}R^3(\rho - \rho_{gas}) \]
\[ F_{vy} = -12\pi\eta R\dot{y} \]

where \( \rho_{gas} \) is the density of gas inside a bubble.

As described in Section 2.3, the direction of the bubble trajectory also depends on the bubble resonance frequency which is given by \[ f_{res} = \frac{1}{2\pi R_0} \sqrt{\frac{3\gamma P_0}{\rho} \left(1 + 2\frac{\sigma}{P_0 R_0}\right) - \frac{2\sigma}{R_0^2 \rho}} \]

If the acoustic standing wave field is weak and oscillating at a frequency above the bubble resonant frequency, bubbles are pushed to the pressure node, otherwise they travel to the pressure antinode.

4. Results and discussion

In this section, the experimental results of bubble behavior in an acoustic standing wave field are presented and are compared with results predicted by the theoretical models. Firstly, a pressure profile measured by the hydrophone is compared with the 1D model prediction. Secondly, comparisons between the bubble trajectories which were observed by the high speed camera and simulated results from the theoretical models are given. Thirdly, results of bubble behavior when they are close to the glass plate are presented. The values of the physical parameters used in this study are \( f = 108 \text{ kHz}, \rho = 1000 \text{ kg/m}^3, P_0 = 101.3 \text{ kPa}, c = 1480 \text{ m/s}, \sigma = 0.072 \text{ N/m}, \gamma = 1, \eta = 0.001 \text{ Pa s} \). Since the investigation of bubble trajectory only requires the recording of bubble motion as a whole rather than observing the oscillation of every driving cycle, a frame rate of 10,000 frames/s was used and was verified to be suitable for this case. On the other hand, a frame rate of 150,000 frames/s was used to study the bubble behavior near the glass plate because a detailed analysis of bubble oscillation shape is desirable for this case.

4.1. Validation of the 1D model

The setup shown in Fig. 3 was used to validate the predictions obtained from the 1D model. As bubbles only move in the water layer, the focus of the validation is mainly on the pressure profile in the water column. Fig. 5 shows the comparison between the calculated impedance of the test cell without the glass plate and that of the cell with the glass layer. It can be seen from Fig. 5 that the addition of the glass plate shifts the resonance frequencies of the structure down to lower frequencies.

A measured pressure distribution in the water along the \( x \) axis (as shown in Fig. 2) and a simulated one at 108 kHz are shown in Fig. 6. The input signal amplitude was chosen as 1.8 V (peak). A pressure amplitude maximum (11.4 kPa) and an amplitude minimum (3 kPa) are located at \( x = 1.7 \text{ mm} \) and \( x = 5 \text{ mm} \) respectively as indicated in the figure.

Fig. 10. Four photo images of an experimental video result. The trajectory of a bubble moving from the injection point towards the glass plate at 108 kHz. The bubble radius was 100 \( \mu \)m and the pressure amplitude was 9.6 kPa. A scale bar indicating a 200 \( \mu \)m length in the images is displayed. (a) at 0 ms; (b) 72 ms; (c) 164 ms; (d) 253 ms.
in the water layer (Fig. 7). The pressure profiles were measured in the
y axis and
wards the glass plate is shown in Fig. 10. The radius of the bubble

4.2. Bubble trajectory in the water layer

The trajectory of a bubble moving from the injection point to-
wards the glass plate is shown in Fig. 10. The radius of the bubble
was 100 µm and the driving pressure amplitude was 9.6 kPa. The
bubble followed a curved path towards the glass plate due to radi-
ation and buoyancy forces exerted on the bubble.

4.2.1. Large bubble

Simulated trajectories are compared with the experimentally
obtained ones (square dotted line) in Fig. 11. Good agreement is
found between the results obtained from the experiment and the-
oretical simulations over the whole experiment. It may be argued
that it would be elegant to show several repeatable test results
rather than one trajectory for each pressure amplitude. However,
the bubble size varies each time due to the coalescence of bubbles
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9.6 kPa to 8.64 kPa in this case. Similar effects can also be seen in Fig. 11b1 and c1 where the experimental results are well predicted by trajectories at 10% lower pressure amplitude than the calibrated one.

It is arguable that the drag force used in the present study may be different from the experiment and therefore contributes to the discrepancy between the bubble moving trajectories predicted by the theory and that of the test. The drag forces in Eqs. (15) and (18) are valid for estimating the dissipative force in the asymptotic limit of high Reynolds numbers. Mei et al. [51] proposed an empirical drag law that matches the asymptotic limits of high and low Reynolds numbers. It was found that the use of Mei’s drag law

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**Fig. 12.** Bubble trajectories at different pressure amplitudes and the influences of pressure amplitude and bubble size on the bubble trajectory. (a1) for a small bubble (bubble radius = 20 μm), the pressure amplitudes applied are 15.36 kPa (−−−), 12.8 kPa (−), 11.52 kPa (−−), 10.24 kPa (−−), and 8.64 kPa for the experimental result (●); (a2) at 12.8 kPa, bubble radii are 20 μm (−−−), 10 μm (−−−), 5 μm (−−−), and 20 μm for the experimental result (●); (b1) for a small bubble (bubble radius = 20 μm), the pressure amplitudes applied are 23.04 kPa (−−−), 19.2 kPa (−−−), 17.28 kPa (−−−), and 15.36 kPa (−−−) for the experimental result (●); (b2) at 19.2 kPa, bubble radii are 20 μm (−−−), 10 μm (−−−), 5 μm (−−−), and 20 μm for the experimental result (●); (c1) for a small bubble (bubble radius = 20 μm), the pressure amplitudes applied are 26.88 kPa (−−−), 22.4 kPa (−−−), 20.16 kPa (−−−), 17.92 kPa (−−−), and 22.4 kPa for the experimental result (●); (c2) at 22.4 kPa, bubble radii are 20 μm (−−−), 10 μm (−−−), 5 μm (−−−), and 20 μm for the experimental result (●). Driving frequency = 108 kHz.

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**Fig. 13.** A small bubble changed its migrating direction after merging with another bubble at 108 kHz. Input pressure amplitude was 4.8 kPa. A scale bar indicating a 200 μm length in the images is displayed. (a) 0 ms; (b) 33 ms; (c) 53 ms; (d) 95 ms; (e) 159 ms; (f) 259 ms.
can hardly change the bubble moving trajectories but is able to shorten the time for the bubbles to move from the injection point to the target. The traveling time obtained from the test, for example, for the bubble of radius of $1.67 \times 10^{-7}$ m and driven at $19.2$ kPa, is about $100$ ms, which is closer to the result (90 ms) predicted by Le- vich’s drag law than that obtained from Mei’s empirical Eq. (50 ms). The Levich’s drag force is therefore used for all the calculations in this paper. Furthermore, as the bubble traveling time is sensitive to the changes of drag force, our test cell could be used for testing the effects of drag force on the bubble motion. However, the discussion of that topic is beyond the scope of the present study and more investigations will be carried out in the future.

As the bubble trajectory not only depends on the pressure amplitude in the water but also on the bubble size, the influence of bubble size on the bubble trajectory are shown in Fig. 11a2, b2 and c2. It can be seen from Fig. 11a2, for example, that increasing bubble radius at a fixed pressure amplitude (9.6 kPa) from $70 \mu m$ up to $130 \mu m$ results in the increase of the height of bubble trajectory at $x = 4$ mm from $0.42$ mm to $3$ mm. The original experimental result and the simulated one (for the bubble of radius of $100 \mu m$) are also shown in the same plot for comparison.

4.2.2. Small bubble

Bubbles of size smaller than their resonance size, on the other hand, are forced to move towards the pressure anti-node. Fig. 12a1, for example, shows that a bubble of radius of about $20 \mu m$ was forced to move from the injection point to the pressure anti-node at $12.8$ kPa. Good agreement is also found here in Fig. 12a1 between the experimental result (square dotted line) and simulated one (solid line). The discrepancies between the theoretical predictions and experimental results can be attributed to the possible deviations in the measurement of pressure profile by the hydrophone and in the calibration of bubble size in our test. Similar to the large bubble case, the influences of pressure amplitude and bubble size on the bubble trajectory are shown in Fig. 12. On the one hand, the bubble trajectories are almost the same, for example, when the pressure amplitude is increased from $10.24$ kPa to $15.36$ kPa as shown in Fig. 12a1. On the other hand, varying bubble radius from $5 \mu m$ up to $20 \mu m$ at $12.8$ kPa (Fig. 12a2) also has limited effects on the bubble trajectories.

Moreover, it has been observed that the small bubble trajectory is not constant especially when the coalescence of small bubbles occurs along the journey. Small bubbles can combine with other ones and start to migrate towards the glass plate instead of the pressure anti-node when the size of the newly formed large bubble becomes large enough. As seen in Fig. 13, a bubble of initial radius of $17 \mu m$ moved towards the pressure anti-node before merging with another bubble to generate a new large one (bubble radius = $38 \mu m$). The new bubble immediately reversed its moving direction and started to travel towards the glass plate. The input pressure amplitude was $4.8$ kPa. Previous studies [29,47] have shown that the change of small bubble trajectory is due to the fact that the Bjerknes force changes its sign when the bubble size is larger than its resonance size. Coalescence processes, however, are not included in our bubble behavior simulations.

4.3. Bubble behavior near the glass plate

To investigate the influence of bubble size on its oscillation mode, the high speed camera was adjusted to focus on a bubble-glass contact area at $84.2$ kPa. A striking difference of bubble oscillation mode is seen from Fig. 14 for two bubbles of different sizes. A more active oscillation was observed for a bubble of size (bubble radius = $40 \mu m$) close to the resonance size. Since this bubble was excited at a higher oscillation mode, the bubble surface instabilities are seen in Fig. 14. On the other hand, a bubble of size (bubble radius = $100 \mu m$) much larger than the resonance size maintained a stable shape when moving towards the glass plate.

It is worth to note that the vigorous microstreaming triggered by oscillating bubbles with surface instabilities is of more interest in applications such as sonoporation [52] and ultrasonic cleaning since the shear stress on surfaces resulting from the microstreaming may generate physical and biological effects within a short range of the oscillating bubbles. The shear force that is transmitted onto cell walls, for example, can temporally open breaches in cell membranes, but should be controlled under certain conditions to avoid fatal damage on the target. The test cell can be used to study the bubble behavior near a surface in the future. The effects of the microstreaming generated from an vigorously oscillating bubble on a target surface and how the influential factors (pressure
amplitude and bubble size) would change the surface instabilities of an oscillating bubble will be studied in more detail.

5. Conclusion

This paper shows the feasibility of using a multi-layered resonator to transport bubbles to a target region and to investigate the bubble behavior near a boundary. The designed test cell, which creates a one-dimensional uniform pressure field across the cross section of the whole stack, is ideal for testing the effects of an acoustic field on the bubble motion, which can be manipulated in a very controlled manner since the pressure field can be accurately predicted by a 1D matrix model.

As the pressure field within the test cell can be accurately quantified, it was possible to use the test cell to test the theory of bubble motion (e.g. modified Keller–Miksis equation and Newton’s second law) and predict the influence of factors, such as different pressure amplitudes or different bubble sizes, on the bubble translational trajectory. Good agreement was found between the theory and the experiment. It was observed, on the one hand, that the trajectories of bubbles driven above resonance were strongly influenced by the pressure amplitude and bubble size. Both an increase of pressure amplitude and a decrease of bubble size forced the bubbles to arrive at the target plate at lower heights. On the other hand, the trajectories of bubbles driven below resonance stayed constant and only little differences of the bubble trajectories were found as a function of pressure amplitude or bubble size. Moreover, as the traveling time of bubbles moving from the injection point to the target plate is sensitive to the changes of drag force, it is possible to use the test cell to quantify the drag force and study its effects on the bubble motion. This was only illustrated in this paper but will be comprehensively studied in the future.

With the test cell, it was also possible to study the bubble behavior near a surface in more detail. At higher pressure amplitude, a bubble of size near the resonance size was more likely to have surface instabilities. The microscreaming from these surface instabilities may contribute to several bubble induced biological or physical effects, such as sonoporation and ultrasonic cleaning. To further the understanding of bubble behavior near a boundary, more investigations will be carried out in the future.

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