Experimental observation of chaotic phase synchronization of a periodically modulated frequency-doubled Nd:YAG laser

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We present experimental observations of chaotic phase synchronization of a sinusoidally modulated frequency-doubled Nd:YAG laser. Periodic and chaotic phase synchronization is detected using recurrence analysis and pseudo ensemble averaging applied to the observed intensity fluctuations and the modulated pump current. © 2009 Optical Society of America

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In 1996 Rosenblum et al. [1] presented the first examples of phase synchronization of coupled chaotic oscillations. Since then, several experimental examples of phase synchronized chaotic systems have been found [2–5], including coupled lasers [6] or driven (modulated) lasers [7]. To detect (chaotic) phase synchronization experimentally, data analysis methods are required that are applied to pairs of signals from both coupled systems. In the most common approach, first phases $\phi_1$ and $\phi_2$ are extracted from the observed time series using polar coordinates in a two-dimensional state space reconstruction (via delay embedding [2] or by means of a Hilbert transform [1,3]). With phase synchronization the difference of these phases is bounded, i.e., there exist integer numbers $n,m \in \mathbb{N}$ such that $|n\phi_1 - m\phi_2| < \text{const}$, although the amplitudes of the oscillations of the coupled systems remain different and may fluctuate chaotically and uncorrelated.

In this Letter, we present evidence for phase synchronization of a periodically driven chaotic laser system consisting of a compact frequency-doubled solid state laser suffering from output instabilities (main frequency $\approx 1$ MHz, system latency $\approx 0.45$ µs) if the pump current exceeds some critical threshold. The underlying chaotic dynamics is caused by the nonlinear coupling of modes being active in two polarization directions. The occurrence of chaotic intensity fluctuations is also called the green problem [8–13] because it limits the usage of frequency-doubled lasers in applications where a constant light output is required. Methods for stabilizing this class of frequency-doubled solid state lasers have been suggested in [14–17] and in the references cited therein. Here we shall focus on another dynamic feature: synchronization of lasers with periodically modulated pump currents.

For the detection of (chaotic) phase synchronization we use the experimental setup shown in Fig. 1. The laser is pumped constantly by a current source where the pump current can be modulated by a generator via a bias-T. The effect of the additional modulation can be measured optionally using one of the two non-frequency-doubled infrared intensities (1064 nm, two orthogonally polarized directions) or the frequency-doubled green intensity (532 nm) that is used in the following analysis.

Figure 2 shows a typical example of phase synchronization between the periodic pump current (drive) and the green intensity (response), where the amplitude is chaotically fluctuating but the phase is locked to the driving current signal. To confirm phase synchronization of such a pair of signals one may check coincidence of main frequencies of the driving and responding system and look for a bounded phase difference. The latter, however, requires that one explicitly extract phase variables from the observed time series. If the reconstructed attractor resembles a spiral around some empty hole in the embedding space a phase can be defined in the form of the angle of rotation $\alpha$ [1,2]. If this is not the case (as with our laser intensity signal) several other methods for detecting locked phase relations have been suggested [2–5], in particular for periodically driven systems [20].

Here we shall present the application of two methods for detecting phase synchronization. The first method was introduced by Romano et al. [19], who showed that recurrence analysis of (chaotic) signals can be used for detecting phase synchronization in cases where no direct phase information is available. Beginning with a sufficiently high-dimensional delay embedding (here $d=3$) of the laser intensity signal (and of the generator dynamics) a recurrence plot...
Fig. 2. (Color online) Time series showing an example of chaotic phase synchronization (top, laser intensity; bottom, pump current modulation).

shows the times $\tau$ for which a considered trajectory in embedding space returns to a well-chosen vicinity $U_{\epsilon}$ with radius $\epsilon$ around its starting point at $t=t_0$. For these recurrence times $\tau$ the recurrence probability

$$P(\tau) = \frac{1}{N-\tau} \sum_{j=1}^{N-\tau} \Theta(\epsilon - \| x_j - x_{j+\tau} \|)$$

(1)

reaches local maxima. Here the Heaviside function $\Theta(\epsilon - \| x_j - x_{j+\tau} \|)$ detects those points $x_j, x_{j+\tau}$ separated from each other by a distance less than $\epsilon$ (where $\Theta=1$ and 0 otherwise). Since the driving generator signal is periodic the recurrence $P(\tau)$ reaches $P(\tau)=1$ for any integer multiple of the period. For a chaotic signal $P(\tau)<1$ holds and (adjacent) maxima are pairwise different. If the maxima of the driving generator and the responding laser system coincide for the same recurrence times $\tau$ temporal rhythms of both systems have adjusted and the phase-synchronized state is reached. Figure 3(a) shows $P(\tau)$ versus $\tau$ for the signals of Fig. 2. The fact that the maxima of the $P(\tau)$ curves of the laser signal and the pump modulation coincide indicates phase synchronization, and the fluctuations of the height of the maxima of $P(\tau)$ for the laser are due to the fact that the intensity is (still) chaotic. Cases of no phase synchronization can be identified by the technique of recurrences, too. As an example Fig. 3(b) shows a recurrence plot where the laser possesses no pronounced recurrences coinciding with the periodic recurrence pattern of the laser. Figures 3(c) and 3(d) show two-dimensional embeddings of the laser intensity $y(t)$ signal using the real and imaginary parts of the analytic signal $s$ that are given by $\text{Re}(s(t))=y$ and $\text{Im}(s(t))=H(y)$, respectively, where $H$ denotes the Hilbert transform [1,3]. The dots are stroboscopic samples of the trajectories triggered by the generator signal $x(t)$. If no phase relation exists they are scattered on the full attractor [Fig. 3(d)], while phase synchronization manifests itself in a clustering of samples [Fig. 3(c)].

To analyze the parameter dependence of the observed synchronization phenomena, a grid has been scanned consisting of $40 \times 40$ combinations of the generator's amplitude $a$ and frequency $f$. To evaluate the synchronization status the cross-correlation coefficient [19]

$$\text{CRP} = \frac{\langle \tilde{P}_L(\tau)\tilde{P}_G(\tau) \rangle}{\sigma_L\sigma_G}$$

(2)

has been calculated using the zero mean recurrences $\tilde{P}_G(\tau)=P_G(\tau)-\langle P_G(\tau) \rangle$, $\tilde{P}_L(\tau)=P_L(\tau)-\langle P_L(\tau) \rangle$, and the standard deviations $\sigma_L, \sigma_G$ of $P_G(\tau)$ and $P_L(\tau)$, respectively. If a synchronized state is reached Eq. (2) attains a high value. If $P_G(\tau)$ and $P_L(\tau)$ do not possess maxima for the same recurrence time $\tau$ the value of the cross-correlation CRP is low. Figure 4(a) shows the correlation coefficient (2) versus the generator's frequency and amplitude. Synchronization occurs in the Arnold tongue close to generator frequencies of $f \approx 1$ MHz.

The second method we used for detecting phase synchronization is applicable to periodically driven systems only and is based on ensemble averaging. In the presence of phase synchronization any ensemble of individual chaotic systems driven by a common periodic signal will generate a large average output, whereas loss of synchronization results in a weak mean field owing to incoherent superposition of individual oscillations [20]. However, the required ensemble of copies of the system of interest is often difficult to obtain. Therefore, here we used for averaging a pseudo ensemble that consists of different trajectory segments of the same driven (laser) system. From the periodic drive signal $x(t_n)$ (here: generator) a phase $\phi(t_n) \in [0,2\pi)$ is extracted for all $t_n=nt_s$ with $n=1, \ldots, N$, during the observation period $T$, where $N$ samples are measured. After subtracting the mean from the response signal $y(t_n)$ (here: laser intensity) these phases are used to sum up (integrate) the response signal such that $y$ values corresponding to the same phase values $\phi$ are added. Technically this superposition fulfilling phase relations is done by divid-
Chaotic phase synchronization has been experimentally observed for a (compact) chaotic frequency-doubled Nd:YAG laser driven by a sinusoidal signal. To detect synchronization two methods were employed: recurrence analysis and (pseudo) ensemble averaging. Both methods provide clear evidence for chaotic phase synchronization for modulation frequencies close to 1 MHz. This synchronization effect could be used to increase the superimposed common output (ensemble average) of several lasers driven by the same periodic pump modulation.

References